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An Energy Model of Plain Knitted Fabric

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ABSTRACT

A new mathematical model describing a plain knitted fabric is proposed in this paper. One major feature of this model is that the yarn in the fabric can be naturally curved with nonlinear mechanical properties. The new model is able to describe the dimensions and also the low stress mechanical properties of a plain knit. Based on an energy analysis, the inadequacy of the classic k-values is explained. Complex dimensional behavior and problems associated with relaxation of knitted fabric are discussed. A more precise prediction of fabric dimensions is possible by including the degree of set as one of the parameters. The paper also explains why most of the studies of knitted fabric dimensional properties in the past have been empirical.

Fabrics knitted from cotton are a very popular, and thus very important to the textile industry. Cotton, being a nonthermoplastic fiber, is unable to be heat set. This category of knitted fabrics will relax naturally after knitting, resulting in changing fabric dimensions. Due to the nature of the knitting process, the fabric is knitted under high stress and extension. Therefore, grey cotton fabrics off the machine will exhibit large and varied amounts of shrinkage. Measurements of shrinkage in grey fabrics are therefore of little value to fabric finishers. What is of value is the reference state of the fabric. This reference state acts as a target of the norm for assessing fabric dimensional behavior.

In attempts to understand the dimensional behavior of knitted fabrics, the key element is the geometry of the knitted loop. Pierce [21], Shinn [27], Leaf [13, 14, 15], Doyle [4, 5], Munden [20], Postle [22, 23], and recently Demiroz *et al.* [3] have all significantly contributed to the geometric analysis of plain knitted fabrics. In particular, Leaf's geometric model [15] has aroused the interest of composite engineers [24]. The success of that model is due to its simplicity and a good description of the actual fabric. Recent innovative work [3] on loop geometry has used spline curves to represent the loop, which is especially useful in the visual display of knitted fabrics on a CAD system.

Yarn jamming in a knitted fabric has long been identified as a major factor determining its the dimensional and mechanical properties. Knapton *et al.* [11] concluded that the stability of a cotton loop is reached when yarn bulking is restricted by yarn jamming.

Alternatively, the dimensional properties of knitted fabrics were studied by some researchers [8, 23, 25] using the force method. In the theoretical models of Postle *et al.* [23], Shanahan *et al.* [25], and Hepworth *et al.* [8], yarn was treated as an elastica [18] that is naturally straight. MacRory et al. [19] and Hepworth [9] attempted to tackle the biaxial load-extension problem of knitted fabrics. MacRory's model emphasized slippage between loops and the biaxial load case with the loop elements being straightened, while Hepworth's model concentrated on the effect of yarn jamming.

Extensive experimental works have been accomplished by Heap *et al.* in the STARFISH project [7]. One of their objectives was to predict the dimensional changes of finished knitted cotton fabrics of some selected structures based on the knitting parameters and finishing route. In their predictions, the dimensional changes were measured against the so-called reference state of a fabric. This very important reference state was assumed to be reached by a vigorous relaxation process of five wash and tumble drying cycles. Heap *et al.* noted that further relaxation after five cycles might still result in further dimensional changes, even though these changes might be small. In practice, there is no measure of how close the five wash cycle state is to the reference state. A theoretical model will be a solution to the situation such that a theoretical reference state can act as a guide to the effectiveness of relaxation processes. However, the model must be relatively simple and able to interface with the fabric actually knitted by a machine. Aiming at this direction, we have developed a new energy model of plain knitted fabrics as described later in this paper.

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Model Descriptions and Assumptions

The new knitted fabric mechanical model we propose is an energy model with sufficient degrees of freedom for loop deformation, including changes in loop height and width. The yarn forming the loops is perfectly elastic and incompressible with zero friction. It is naturally curved, and its paths in the deformed states are the elements of a prescribed family of space curves.

This model can be used to determine the biaxial tensile behavior of a plain knitted fabric. The yarns can be axially compressed or extended at the low load region. Conditions of loop jamming are taken as the geometric constraints of loop deformation.

LOOP GEOMETRY OF THE NEW MODEL

Studying knitted fabric mechanical properties with the energy method requires a well-defined loop geometry. Obviously, a simple loop geometry can ease the complexity of the solution. On the other hand, the assumed loop geometry must have enough degrees of freedom to allow the fabric to deform with a close approximation to the actual loop shape.

As shown in Figure 1, the right-hand orthogonal (xyz) axes are adapted with the x-axis in the course direction, the y-axis in the wale direction, and the z-axis in the direction normal to the fabric plane. When considering the left half of a loop, by observing that the x-coordinate of a point along the loop with increasing length increases, then decreases and increases again, a polynomial with a minimum degree of three is required. For the y-coordinate, it increases slowly, then sharply and slowly again, with zero rate of change at both ends. A cosine function with a half cycle is appropriate. For the z-coordinate, it increases up to a maximum and then decreases with zero rate of change at both ends and midpoint. Again, a cosine function with a full cycle is required. The cosine function has been used by Leaf *et al.* [15] to specify the variation of the z-coordinate of the loop.

The parametric representation of the central axis of the loop is

$$x(t) = at^3 - 1.5at^2 + 0.5(a + w)t \quad , \tag{1}$$

$$y(t) = 0.5(c + 2e)(1 - \cos \pi t) \quad , \tag{2}$$

$$z(t) = 0.5(t_h - d)(1 - \cos 2\pi t) \quad , \tag{3}$$

where w is the loop width, $(c + 2e)$ is the loop height, e is the adjacent loop overlapping distance, t_h is the fabric thickness, d is the yarn diameter, a is a free parameter, and t is the parameter of the space curve, *i.e.*, $t \sim [0, 1]$. Later, a, e, and t_h can be determined by the condition of the loop touching in the x, y, and z directions.

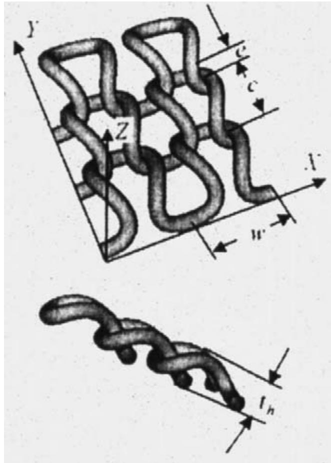

FIGURE 1. Geometry of plain knit model.

LOOP SEPARATION PARAMETER χ

With reference to Figure 2, the parameter t takes the values β_1, 0.5, β_2 at points A, B, and C, respectively. Because the knitted loop is assumed to be antisymmetric about point B, we have

$$x(\beta_1) + x(\beta_2) = w/2 \quad . \tag{4}$$

Consider loop touching between C and A′,

$$x(\beta_1) - x(\beta_2) = \chi d \quad , \tag{5}$$

with the assumption of incompressible yarn,

$$\chi \geq 1 \quad . \tag{6}$$

Consider loop touching between A and D,

$$x(\beta_1) \leq w/2 - d/2 \quad . \tag{7}$$

Together with Equations 4 and 5, we have

$$\chi \leq \frac{w}{2d} - 1 \quad . \tag{8}$$

In summary,

$$1 \leq \chi \leq \frac{w}{2d} - 1 \quad . \tag{9}$$

When χ is within this range, loops do not touch each other in the x-direction.

DETERMINING SHAPE PARAMETER a

Assumption: Points of the loop touching (at A and C in Figure 2) are the turning points in Equation 1, *i.e.*,

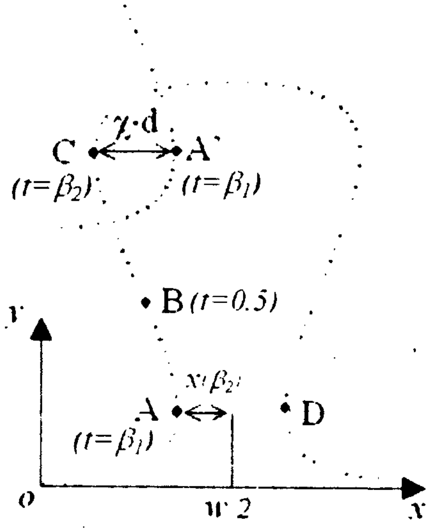

FIGURE 2. Loop touching.

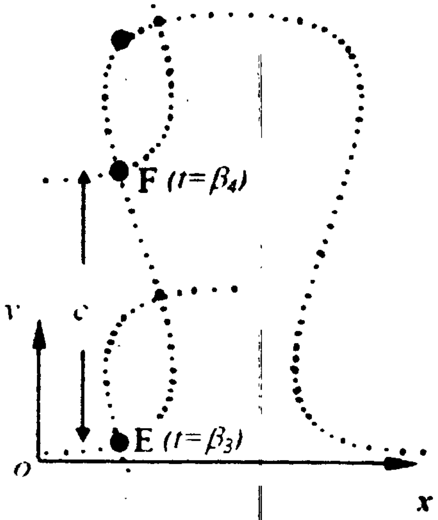

FIGURE 3. Loop touching in z-direction.

$$dx/dt = 0 \Rightarrow 3at^2 - 3at + 0.5(a + w) = 0 \quad . \qquad (10)$$

If τ_1 and β_2 are the roots of Equation 10, we have

$$\beta_1 = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{a-2w}{3a}}, \quad \beta_2 = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{a-2w}{3a}} \quad , (11)$$

with $x(\beta_1) - x(\beta_2) = \chi d$, $0 \le \beta_1 < \beta_2 \le 1$. In this way, the shape parameter a can be expressed in terms of χ.

DETERMINING LOOP OVERLAP e

In the knitted fabric structure, loop interlocking results in yarn overlapping. This overlapping space is required for determining course spacing, as described by the earlier assumption. Based on the loop geometry of the new model as described in Equation 2,

$$e = y(\beta_1) = 0.5(c + 2e)(1 - \cos \pi\beta_1) \quad . \qquad (12)$$

After simplification, we have

$$e = \frac{c}{2}\left[\frac{1}{\cos \pi\beta_1} - 1\right] \quad . \qquad (13)$$

DETERMINING FABRIC THICKNESS t_h

Points E and F (at $t = \beta_3$ and $t = \beta_4$ as shown in Figure 3) are the points in the loop at which the loops touch each other in the z-direction (thickness direction) and would satisfy the two conditions. They have a height difference equal to c and they have equal x-coordinates, *i.e.*, $y(\beta_4) - y(\beta_3) = c$ and $x(\beta_3) = x(\beta_4)$. From the two equations, we can find β_3 and β_4.

Fabric thickness t_h can be determined by the relation $z(\beta_4) - z(\beta_3) = d$; after simplification, we have

$$t_h = \left[\frac{2}{\cos 2\pi\beta_3 - \cos 2\pi\beta_4} + 1\right]d \quad . \qquad (14)$$

DETERMINING SEPARATION PARAMETER χ

The loop separation parameter can be determined by maintaining an unchanged loop length in the deformed fabric, *i.e.*, $L_y = L_{yo}$. When χ satisfies the following condition,

$$1 \le \chi \le \frac{w}{2d} - 1 \quad , \qquad (15)$$

the yarn loop remains unstrained. Otherwise jamming occurs, the yarn is extended or compressed, and yarn strain e_y is

$$e_y = L_y/L_{yo} - 1 \quad . \qquad (16)$$

Mechanical Energy of a Deformed Loop

The total mechanical energy U of one yarn loop is equal to the sum of the tensile energy U_t, bending energy U_τ, and torsional energy U_κ, *i.e.*, $U = U_t + U_\tau + U_\kappa$:

$$U_t = \frac{E\,\text{tex}}{2}\int_0^{L_y} e_y^2 ds \quad ,$$

$$U_\tau = \frac{C}{2}\int_0^{L_y} (\tau - \tau_o)^2 ds \quad ,$$

$$U_\kappa = \frac{B}{2}\int_0^{L_s} (\kappa \sin \phi)^2 + (\kappa \cos \phi - \kappa_o)^2 ds \quad .$$

Yarn extension is assumed to be constant along the yarn, and with the re-parametrization in t, the total mechanical energy in dimensionless from U' is given below:

$$U' = \frac{E \text{ tex } e_y^2 L_y^2}{2B} + L_y \int_0^1 [(\kappa \sin \phi)^2 + (\kappa \cos \phi - \kappa_o)^2 + N(\tau - \tau_o)^2] \frac{ds}{dt} dt \quad , \quad (17)$$

where $U' = UL_y/B$, tex and L_y are the yarn linear density and loop length, respectively, E, B, and C are the tensile, bending, and torsional moduli of the yarn, $N = C/B$, ϕ (phase angle) is the angle between the yarn's principal axes and the principal normal to the yarn axis, κ and τ are the total curvature and torsion of the yarn, and κ_o and τ_o are the total curvature and torsion of the yarn in its natural state.

Biaxial Tensile Model of Knitted Fabric

The external forces acting on the knitted fabric are the tensile loads in the course and wale directions, T_c and T_w (in force per course and per wale, respectively), as shown in Figure 4. If the external forces are conservative, they have a potential W. Since no frictional forces are included in the analysis and the yarn is assumed to be elastic, the knitted fabric is treated as a conservative system, and the potential energy V of the fabric is the sum of the elastic strain energy U and the total work done W by external forces. The total potential energy is

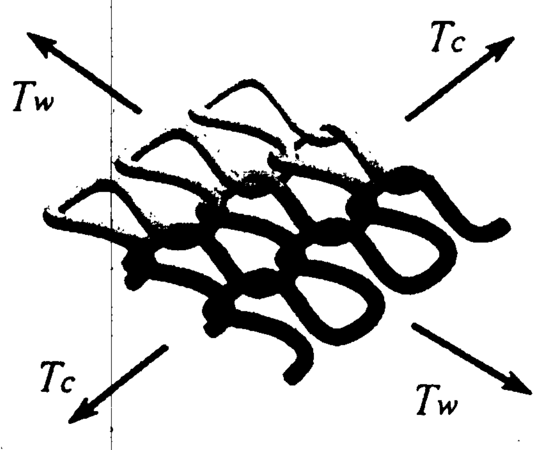

FIGURE 4. Fabric under biaxial loading.

$$V(c, w) = U(c, w) + W(c, w) \quad .$$

By the principle of minimum potential energy [12], with prescribed external loads (T_c and T_w), the fabric is in stable equilibrium if and only if the total potential energy $V(c, w)$ is at a minimum. A necessary and sufficient condition for stable equilibrium is $\delta V = 0$ and $\delta^2 V > 0$, where V is the total potential energy of the fabric:

$$\delta V = \frac{\partial V}{\partial c} \delta c + \frac{\partial V}{\partial w} \delta w \quad , \quad (18)$$

where δc stands for an arbitrary quantity of c, etc.,

$$\delta V = 0 \Leftrightarrow \begin{cases} \dfrac{\partial V}{\partial c} = 0 \Leftrightarrow \dfrac{\partial U}{\partial c} = T_w \\ \dfrac{\partial V}{\partial w} = 0 \Leftrightarrow \dfrac{\partial U}{\partial w} = T_c \end{cases} \quad . \quad (19)$$

When the external forces are prescribed, the fabric dimensional parameters (c and w) can be determined from this system of partial differential equations. Hence, we have a biaxial tensile model of knitted fabric.

Degree of Set

In this energy model, the yarn forming the knitted loop is naturally curved. This natural curvature is defined as the curvature of a yarn course when removed from the knitted fabric without introducing any disturbance to the yarn. Instead of discussing the practical difficulties involved in measuring natural curvature, we explain the concept as follows:

It is very rare that a yarn course is found to be straight after being unraveled from a finished fabric. For a freshly machine-released fabric, the yarn may be much less curved than that of the finished fabric, but it still looks wavy. As in the knitting process, the yarn is bent into loops with large curvature, especially near the needle loop and sinker loop. Plastic deformation may take place due to fiber slippage and/or fiber permanent extension near the outside region of the curved yarn. The finishing process in most cases has a certain setting effect on the yarn in the knitted fabric. As a result, the yarn that is removed from a knitted fabric should have a curvature value between zero and the curvature at the corresponding position in the knitted loop. The two extremes stand for naturally straight yarn and fully set yarn.

Let $\kappa(s)$ be the curvature of the yarn in the fabric and $\kappa_o(s)$ its natural curvature. When $\kappa_o(s) \equiv 0$, the yarn is naturally straight. When $\kappa_o(s) = \kappa(s)$ and $\tau_o(s)$

$= \tau(s)$ for $s \in [0, L_y]$, the yarn is fully set. By assuming that $\kappa_o(s)/\kappa(s) = \tau_o(s)/\tau(s)$, the degree of set ($\psi$) of the yarn is defined as $\psi(s) = \kappa_o(s)/\kappa(s)$, and $0 \le \psi(s) \le 1$.

EFFECT OF DEGREE OF SET

The energy analysis of the plain knit structure in this work considers the degree of set in the yarn after it has been knitted into a fabric. This is important because the degree of set is critical in determining the total energy in the structure. When the fabric is fully relaxed, the internal energy is at its minimum. As shown in Figure 5, the loop model proposed here predicts that when the degree of set is 1.0, the minimum energy will be zero. When the yarn has a natural shape that differs from its shape when it is in the fabric, the degree of set is less than 1.0. When the elastic properties of the yarn take effect, the minimum energy is >0. The minimum energy value increases with a decrease in the degree of set, and it reaches a maximum value when the degree of set is 0.0 such that the natural shape of the yarn is straight even after being held in the fabric for a prolonged time.

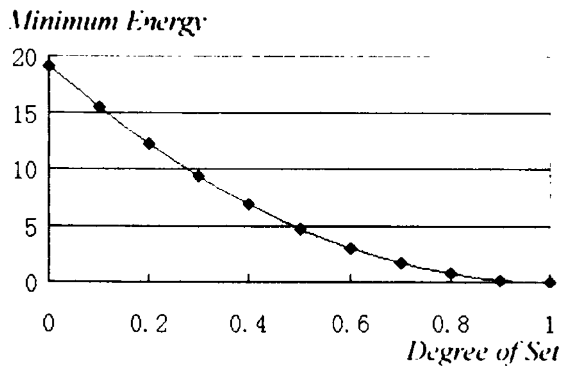

FIGURE 5. Minimum energy values for fabrics with yarns of different degrees of set

The fully relaxed fabric dimensions are also affected by the degree of set. When a fabric is fully relaxed, its loops will take a certain shape. But the prediction of the loop model shows that in the fully relaxed state, the course spacing and wale spacing sets are different for different degrees of set in the yarn, as illustrated in Figure 6. The ratio between course spacing and wale spacing is known as the loop shape factor, and it is directly related to the length and width of the whole fabric.

A change in the degree of set in the yarn results in a different energy value and a different loop shape and dimensions when the fabric is fully relaxed. When the

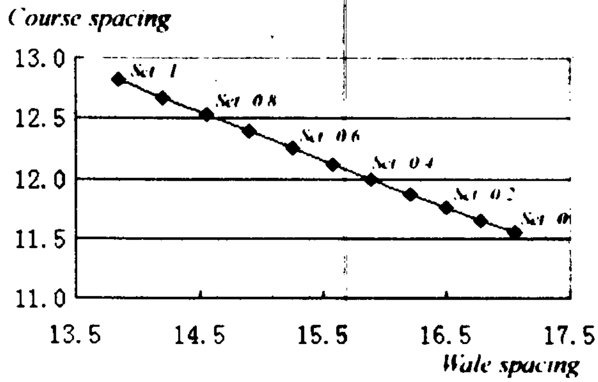

FIGURE 6. Points of minimum energy for fabrics with yarns of different degrees of set

fabric is not fully relaxed, there will be a range of wale and course spacing sets for each particular energy level.

Similarly, when the degree of set in the yarn changes, there will be a new set of wale and course spacings for the same degree of relaxation in the fabric. The dimensions of fabrics with different degrees of set in the yarns will therefore have different dimensions for various degrees of relaxation, including the fully relaxed state.

Dimensions of Plain Knit Fabric

PREDICTING FABRIC DIMENSIONS

Experimental observations by Doyle [4] confirmed the dependence of fabric area on the loop length of plain knit fabrics. The dimensions of a knitted fabric are believed to be predominately determined by loop length and are independent of other yarn and knitting parameters. The relationship can be described by a set of k-value [20]:

$$cpc = \frac{k_c}{L_y} , \tag{20}$$

$$wpc = \frac{k_w}{L_y} , \tag{21}$$

$$S = \frac{k_s}{L_y^2} , \tag{22}$$

where cpc is course per cm, wpc is wales per cm, S is stitch density in stitches per cm^2, and L_y is loop length.

The k-values predict that the loop length is the sole parameter determining the dimensions of plain knitted fabric, and these dimensions can be precisely predicted by controlling the loop length when properly relaxed. However, the k-values known to most researchers are not, in general, universal to all plain knitted fabrics. Different sets of k-values exist for different fabrics and also for the same fabric at different relaxation states. Although experimental studies showed strong linear correlations between the inverse of the loop length and the fabric dimensions (such as coarse density *cpc* and wale density *wpc*), measured data points seldom tied up as expected. The relationship described by $cpc = k_c/L_y$ should result in a straight line passing through the origin when the *cpc* values is plotted against $1/L_y$.

Experiments [6, 16] demonstrated that the linear regression lines more often than not result in significant intercepts, especially in the less relaxed states such dry and wet relaxation. The *k*-values therefore represent only an approximation of the relationship between the dimensional parameters of the fabric and the loop length. As a result, the *k*-value is often less than useful for predicting fabric dimensions due to poor precision. So far, there is not a single set of *k*-values published with full confidence that would be useful for predicting fabric dimensions in the industry and that can be reproduced every time. Most finishing mills are still relying on their own data accumulated over the years. The STARFISH project [7] built up a data base for predicting fabric dimensions instead of using *k*-values. That project included yarn type, fabric structure, and finishing routine as variables, and predictions were made by interpolation or extrapolation of the data sets obtained experimentally. The project assumed a linear relationship between the fabric dimensional parameters and the inverse of the loop length, but permitted a straight line that did not pass through the origin by including a coefficient in the equation:

$$cpc = a_o + \frac{a_1}{L_y} + a_2\sqrt{\text{tex}} \quad , \tag{23}$$

where *cpc* is course density, tex is the linear density of yarn, a_o and a_1 are regression coefficients, a_2 is a regression coefficient related to yarn linear density, and L_y is loop length. Considering the situation [17] where the loop length L_y increases, the *cpc* value will decrease accordingly:

$$\text{If } L_y \to \infty \text{ then } \frac{1}{L_y} \to 0 \text{ and } cpc \to 0 \quad . \tag{24}$$

It would only be logical to expect that the line should pass through the origin. Obviously the STARFISH project, using linear regression equations with intercepts, was just being pragmatic, intending to provide the best approximation within a practical range of fabrics. The *k*-values may only be an over-simplified solution after all.

The inability to arrive at a universally applicable set of *k*-values is assumed to be due to the relationship between the dimensional parameter such as *cpc* or *wpc* and is actually nonlinear in most relaxed states of a fabric [17]. Figure 7 shows the relationship between the inverse of the loop length and the fabric dimensional parameters based on the loop model proposed in this paper. Plain knit fabrics are assumed to be fully relaxed, *i.e.*, at their minimum energy state, at a degree of set of 0.8, and changes in loop shape during relaxation are free from frictional constraints. It is clear that the relationships between the inverse of the loop length and either *cpc* or *wpc* are both nonlinear. The gradient of the lines increases when fabric tightness increases. Also, the rate of gradient change is faster when the value of $1/L_y$ is large or when the fabric is on the tight side.

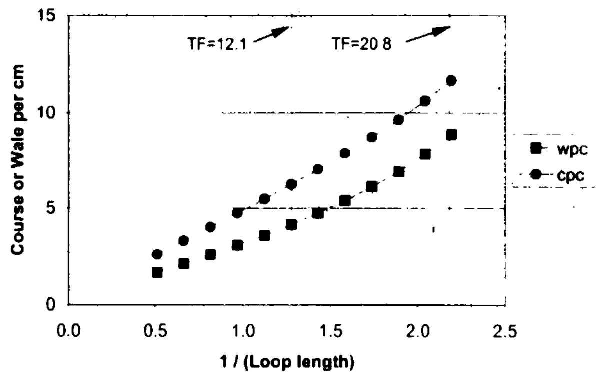

FIGURE 7. Theoretical relationship between *cpc*/*wpc* and $1/L_y$ when fully relaxed. Fabric dimensions at different loop lengths, degree of set = 0.8, yarn tex = 90.

When the fabric tightness factor is given by $\sqrt{\text{tex}}/L_y$, *with* L_y in cm, the practical range is about 10 to 22, 10 being very slack and 22 being very tight. The theoretical fabrics in Figure 7 cover a range of tightness factors from 4.7 up to 20.8, corresponding to $1/L_y$ values 0.5 and 2.2, respectively. The practical range is indicated by the two lines for tightness of 12.1 and 20.8. Examining the data points in this region generated from the loop model, we see that they form a line not much different from a straight line, since the rate of gradient change is low in this region. If the relationship is assumed to be linear to start with, experimental data points in this region will falsely support that assumption, even though the line is in fact not straight. This explains why the STARFISH project prefers to adopt a straight line with an intercept, conflicting with the argument that when $1/L_y \to 0$ then $cpc \to 0$. In that case, they avoid the increased error by forcing the regression line passing through the origin.

Figure 8 shows a comparison of this theoretical prediction with results published by Knapton *et al.* [10]. Their experimental results belonged to a set of fabrics

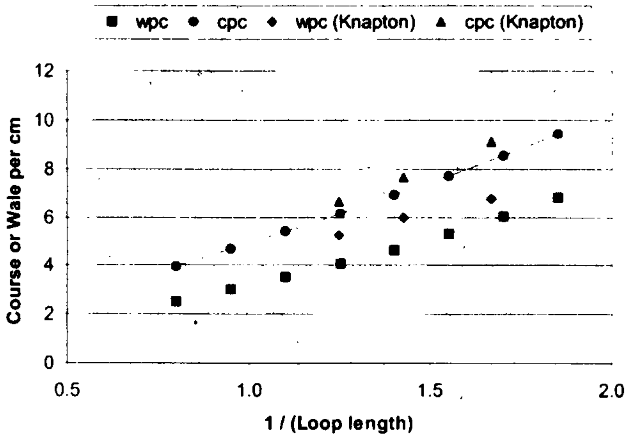

FIGURE 8. A comparison with Knapton's [10] experimental results.

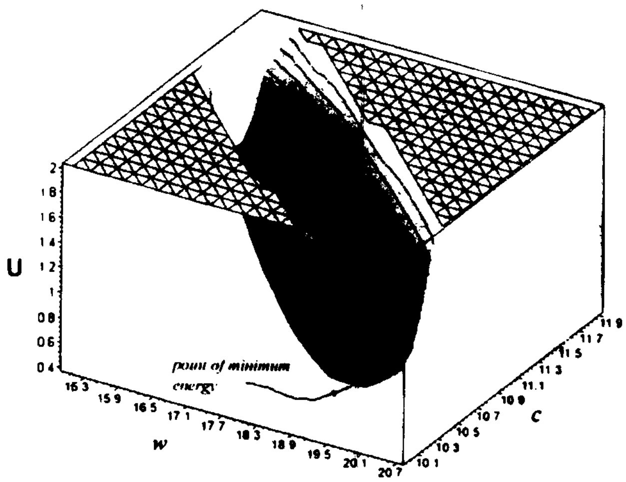

FIGURE 9. Energy mapping of a plain knit fabric. Fabric dimensions at different loop lengths, degree of set = 0.8, yarn tex = 90.

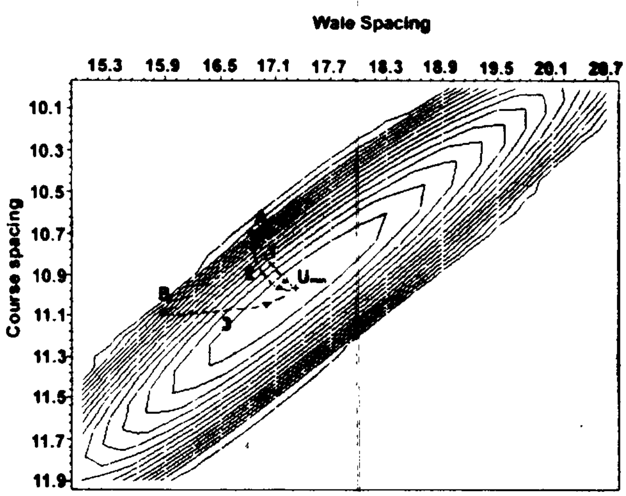

FIGURE 10. Relaxation along energy contours.

tumbled at 70° for 1 hour, taken as the fully relaxed state, since the fabrics would be practically stable in dimensions. The experimental results bunched in a narrow region of fabric tightness, as discussed above. The *cpc* values match very well with the theoretical predictions. The *wpc* values do not match as well as the *cpc* values, but are considered to behave very similarly. Some discrepancy is expected in this case and there should be, since it is highly probable that the experimental fabrics adopted here, though they went through a vigorous relaxation process, were not at their truly minimum energy state. What this set of data points is compared with is a calculated data set based on a theoretical minimum energy state.

RELAXATION AND CHANGE OF FABRIC DIMENSIONS

In practice, achieving a perfect minimum energy state for a fabric is believed to be difficult. According to the energy analysis of our loop model, there would be one loop shape possessing the minimum potential energy, as shown by the energy mapping in Figure 9 (strain energy is equal to total potential energy when external forces vanish). At energy levels above the minimum, there can be loops with different shapes but with the same potential energy. The different loop shapes are evident in the different ratios of course and wale spacing. Figure 10 shows the energy contours of a fabric when taking up different loop shapes, resulting in different course and wale spacings. The contours show that the gradient of the energy levels is steeper when the energy is high. When the energy level is close to the minimum it flattens, meaning that, in agreement with Shanahan's [25] conclusion, the minimum energy state is unstable and difficult to achieve. Experimental observations will therefore mostly involve fabrics with potential energy above the minimum, where a range of loop shapes is possible, forming one of the contours shown in Figure 10.

In the energy change perspective, the mechanism of relaxation is viewed as follows: A fabric at a certain relaxed state possessing a certain energy can be represented by point A, as shown in Figure 10, on one of the contour lines determined by its loop shape. If the fabric is allowed to relax, the point starts to move to the other contour line of lower energy until ultimately arriving at the point of minimum energy U_{min}, as shown by relaxation route 1 in Figure 10. Possibly the path of point A moves from the higher energy contour to the point of minimum energy by an alternate relaxation route 2 (in Figure 10), depending on the environment or relaxation process. On the other hand, the same fabric may also have a different starting point on the energy contour, as illustrated by point B in Figure 10. Clearly, upon relaxation, yet another route 3 different from the previous two will be taken up. The dimensions of the fabric or the loop shape at any state of relaxation before reaching the true fully relaxed state is much affected by the relaxation starting point and the relaxation route down the energy contours.

A fabric in the machine state possesses high potential energy, and when it is allowed to relax, will gradually move to lower energy levels until eventually, in theory, it reaches the minimum energy state. In the course of relaxation, the loop shape changes. For example, when a fabric is moving from the dry relaxed state to the wet relaxed state, the loop shape changes. After relaxation, the loop will take up a new shape, and that will be one of the shapes as defined by the wet relaxed energy contour shown in Figure 10. The different loop shape with the same energy level can be larger when the energy level is high. Consequently, it is not surprising to see inconsistency in experimental observations on the relationship between loop length and fabric dimensions. The problem is more acute when the fabric is less relaxed or at a higher energy level. It is possible for two sets of identical fabrics relaxed by the same process to end up with different loop shapes and hence different fabric dimensions, unless the truly minimum energy state has been reached.

The loop model proposed here predicts that plain fabrics knitted from the same yarn when fully relaxed will take up a definite loop shape, and so the dimensions are determined by loop length and independent of machine and finishing parameters. Unfortunately, in practice fabrics are observed and used at energy states above the minimum. In this case, the dimensions and behavior of these fabrics will be affected by their fiber and yarn properties because there will be different degrees of set for different yarns and fibers. The degree of set of the yarn in the fabric will result in significantly different energy contours and hence loop shapes and fabric dimensions. For a similar reason, machine parameters such as machine gauge, fabric take-down tension, and stretch board width will affect the relaxation starting point on the energy contour. Consequently, the relaxation route will be different. The finishing or relaxation process will also have significant effects on fabric dimensions. Starting with the same grey fabric (same relaxation starting point on the energy contour), the relaxation process will determine the relaxation route down the energy contours. The effectiveness will also determine which final energy contour the fabric will reach and hence the dimensions.

Empirical studies of the relationships between fabric dimensions and yarn, machine, and finishing parameters by multiple regression analysis will have no meaning if the relaxation starting point, that is, the degree of set in the yarn, is not considered.

Model Applications

Besides predicting biaxial tensile properties of knitted fabrics, the model can also be used to solve the dimensional problems, for instance, predicting the reference state of a partially set knit, (*i.e.*, a fabric composed of naturally curved, twist-lively yarn). The initial state of the fabric is not in stable equilibrium. Friction between loops temporarily maintains the shape, but that shape will change with a small disturbance. At the fully relaxed state, all frictional forces vanished since there is no more relative movement. We can determine the fabric dimensional parameters by setting the external forces at zero:

$$\frac{\partial U}{\partial w} = 0 \quad \text{and} \quad \frac{\partial U}{\partial c} = 0 \quad . \tag{25}$$

The accuracy of the predicted results is determined by a number of factors. The most important one is the availability of the natural curvature and torsion of the yarn, which is difficult to measure and changes with time and environmental conditions. Other important factors include the accuracy of the measured yarn mechanical properties (*i.e.*, tensile, ending, and torsion moduli) since they are highly nonlinear and coupled with one another (*i.e.*, bending rigidity decreases when the yarn is untwisted). It is much better to replace the experimentally measured yarn mechanical properties with theoretical models. A recently developed tensile torsional model of singles [1] and two-ply yarn [2] can be used, but there is no suitable yarn bending model the literature. One major reason is that the complicated movement of fibers in yarns during bending deformation can not be simulated satisfactorily. Once this problem is solved, the knitted fabric mechanical properties could be predicted by starting with the fiber properties.

Conclusions

We have developed a new mechanical model for a plain knitted fabric using the energy approach. The geometry of a knitted loop should be as simple as possible and yet capable of describing the dimensional changes in a fabric (*e.g.*, wale/course per unit length, skewness). The degree of set of the yarn in the knitted fabric has a great influence on the fabric's dimensional stability and hence the reference state. In our new model, the yarn in the fabric can be naturally curved. This natural curvature and torsion of the yarn can be input as functions of arc length. The total mechanical energy is calculated based on the difference in bending strain and torsional strain of the yarn with respect to the natural curvature and torsion.

We have explained the inability of the classic k-values to effectively predict fabric dimensions. The problems and inconsistencies of empirical studies of knitted fabric dimensional behavior can also be explained by our theoretical model. The important fact that the yarn in a fabric is partially set must not be overlooked, and only when this parameter is taken into account can the dimensional behavior of a plain knit structure really be understood.

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Appendix

c	course spacing
w	loop width
d	yarn diameter
t_h	fabric thickness
e	adjacent loop overlapping distance
κ	total curvature of a point along the center line of the loop
κ_o	total curvature in the natural state of yarn
τ	torsion of a point along the center line of the loop
τ_o	torsion in the natural state of yarn
U	total mechanical energy
V	total potential energy
W	total work done by external force
r_y	radius of yarn
e_y	yarn strain
L_y	loop length
E	tensile modulus of yarn (in force per tex)
B	bending rigidity of yarn
C	torsional rigidity of yarn
ϕ	phase angle (angle between yarn's principal axes and the principal normal to the yarn axis)
T_c	tensile load in the course direction
T_w	Tensile load in the wale direction
ψ	degree of set

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